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Statistical Verification of the Stefan–Boltzmann Relation Using Observational Stellar Catalog Data

Priti Goyal*

Associate Professor, Department of Physics, Acharya Narendra Dev College, Delhi, India.

***Corresponding author:** pritigoyal@andc.du.ac.in

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Abstract: *The relationship between stellar luminosity, stellar temperature, and stellar radius plays a critical role in comprehending the stellar structure and evolution. In this work, the empirical investigation of stellar luminosity scaling was carried out through the analysis of the stellar catalog data to validate the Stefan-Boltzmann Law. The analysis was carried out to understand the relationship between stellar luminosity and stellar temperature as well as the combined parameter R^2T^4, which theoretically controls the radiative output of the stars. The analysis was carried out through the log-log regression analysis to understand the conformity between the theoretical predictions and the empirical data. The log-log regression analysis between the stellar luminosity and the combined parameter R^2T^4 resulted in a slope of approximately 0.96, which closely matches the theoretical value. Moreover, the comparison between the observed and predicted luminosity values provided a coefficient of determination equal to $R^2 = 0.92$, which shows a high degree of agreement between the theoretical model and observational data. In addition to that, the stellar distribution was examined based on the Hertzsprung - Russell diagram, which shows specific areas that represent the population of main sequence stars, giants, and white dwarfs. Residual analysis based on stellar types shows differences in the accuracy of the theoretical relation during different evolutionary stages.*

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Introduction

It is vital to comprehend the physical properties of stars since the parameters of stars give us insight into the structure, evolution, as well as the mechanisms of production of energy. Among the parameters of stars is the luminosity of the star. Luminosity is the energy emitted by the star per unit of time and is a vital characteristic of the activity of stars. It is closely related to other stellar parameters like the temperature as well as the radius of the star. By understanding the relationship between luminosity and the other parameters of the stars, astronomers have been able to comprehend the physics of the stars [1].

A key tool in stellar astrophysics is the so-called Hertzsprung–Russell diagram (HR diagram), where the connection between stellar luminosity and surface temperature is depicted. It was independently proposed by Ejnar Hertzsprung and Henry Norris Russell in the early twentieth century and remains one of the key tools in understanding stellar classification and evolution [2]. Various types of stars, such as main sequence stars, giants, supergiants, and white dwarfs, are located in different parts of the HR diagram, indicating differences in their internal structure and energy generation mechanisms [3].

The relationship between the luminosity, radius, and surface temperature of stars is given by the Stefan-Boltzmann law. According to this law, the total luminosity of a star is proportional to the square of the radius and the fourth power of the surface temperature of the star. This is the theory upon which the estimation of the luminosity of stars is based and is an important part of the theory of stellar structure [1]. In actual astronomical observations, there may be some deviation from the ideal theory due to measurement uncertainties, stellar composition differences, or evolutionary effects.

Recent advances in astronomical observations and stellar databases have made the detailed analysis of the primary stellar parameters, such as luminosity, effective temperature, and stellar radii possible. The study of these parameters is of great importance in terms of stellar structure, radiation, and evolution, as well as in verifying theoretically derived laws of stellar energy emission.

Past studies have explored the relation between the luminosity, temperature, and radius of stars by using observational data. For instance, Luca Casagrande et al. in their research article [4] proposed a calibrated effective temperature scale for stars. They further explored the relationship between the temperature and luminosity of stars by using photometric data. Another research article by Andrew W. Mann et al. [5] used observational data to calculate the fundamental parameters of nearby stars, such as effective temperature, luminosity, radius, and mass. They further used the Stefan–Boltzmann Law to calculate the values of these parameters. Furthermore, Zeki Eker, et al. [6] extended the mass-luminosity and temperature relations of main sequence stars, using a large sample of well-studied stellar systems, reinforcing the connection between stellar parameters and theoretical predictions. Besides, Leonid M. Martyushev, et al. [7] studied the luminosity-temperature relation of thousands of main sequence stars, showing that observations agree with predictions of stellar physics. Observational studies based on the temperatures and luminosities of the stars are also commonly employed in the formation of the H R diagram, as is evident in the spectroscopic studies [8]. These studies have clearly shown that observational stellar data is a useful tool for the empirical verification of the theories of stellar radiation and structure.

Although considerable progress has been made in the understanding of the relationships between stellar parameters, the majority of the work done in the field mainly focuses on astronomical surveys and stellar modeling. There is still a need for simple empirical analyses that demonstrate the possibility of verifying the fundamental stellar radiation laws using the available observational data. Under these circumstances, the present study seeks to analyze the observational stellar data in order to understand the relation between the luminosity, temperature, and radius of the stars, and perform an empirical verification of the Stefan-Boltzmann Law. The present study has also employed regression analysis and graphical presentation of the data, including the creation of the HR Diagram, in order to understand the stellar populations and compare the results with the theoretical predictions.

Data and Preprocessing

The basis of the analysis in the present study is the publicly available stellar catalog dataset of the observational parameters of stars of different spectral types [9]. The data from the stellar catalog is commonly used in various astrophysical studies due to the systematic determination of the fundamental stellar parameters, including the effective temperature, luminosity, radius, as well as the spectral type of the stars. The use of the large astronomical catalogs of the observational parameters of stars is to investigate the statistical correlations between the stellar parameters or to verify the theoretical relations based on the physics of the stars [10]

The stellar dataset contains information about 240 stars with physical parameters like surface temperature, luminosity, and radius. These physical parameters are of great use to study the luminosity-temperature relation as per the Stefan-Boltzmann law. The dataset also contains classification information like the spectral class and type of the star. These physical parameters are generally used to study the stellar structure and are of great use to analyze the physical relations governing the output of the star. The values of luminosity in the dataset are usually given in units of luminosity relative to the Sun's luminosity. Similarly, the stellar radius is often given in units of the Sun's radius. The effective temperature is given in units of Kelvin. This enables us to compare the data directly to the theoretical relations used in stellar astrophysical studies [2].

Before the analysis, the dataset was checked for missing or inconsistent data. Basic preprocessing of the data was carried out to make the data clean for the results. Data cleaning involved removing the missing data and ensuring that all the parameters were given in consistent physical units.

Due to the range of luminosity in stars of different kinds, the use of logarithmic scaling is appropriate for better visualization of the stellar data and comparison of stellar properties.

In addition to this preprocessing, various quantities were also calculated using known physical relationships in order to test theoretical expectations related to stellar luminosity. It is worth noting that these calculated values were later compared with the values related to luminosity as provided in the catalog in order to test the theoretical expectations against observational data. Such a comparison represents an effective way of testing classical astrophysical relationships against observational data.

The cleaned and processed data set was then utilized to carry out statistical analysis as well as graphical visualization, including the comparison between predicted and actual values of luminosity, as well as the study of the distribution of stars in the HR diagram. This allows a quantitative evaluation of how well actual observations conform to the theoretical relations like the Stefan-Boltzmann relation.

Methodology

The main objective of the present study is to investigate the relationship between the luminosity of stars and their fundamental stellar parameters. The study is based on the theoretical framework of classical stellar physics. Specifically, the relationship between luminosity, stellar radius, and effective temperature is used as the basis for the study. Comparisons between the predicted luminosity values and the actual luminosity values of stars are made to assess the accuracy of the theoretical models.

The fundamental physical relation applied for the analysis is the Stefan-Boltzmann law, which explains the total energy emitted by a star. Based on the law, the luminosity of a star depends on the surface area of the star and the fourth power of the effective temperature of the star. The relation can be written as

$$L = 4\pi R^2 \sigma T^4$$

Where L represents the stellar luminosity, R is the stellar radius, T is the effective surface temperature, and σ is the Stefan–Boltzmann constant. This relation provides the theoretical basis for estimating stellar luminosity from observable stellar parameters [1].

However, in order to perform comparative analysis with respect to other stars as well, the equation is often expressed in solar units by normalizing the parameters with respect to their values in the Sun. This expression is often used in stellar astrophysics to simplify the comparative study of the stars with respect to the Sun. The normalized expression is

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T}{T_\odot}\right)^4$$

Where $L_\odot$, $R_\odot$, and $T_\odot$ represent the luminosity, radius, and effective temperature of the Sun, respectively [2]. Using this relation, theoretical luminosity values were calculated for each star in the dataset based on the observed radius and temperature values.

In order to assess the degree of consistency between the theoretical predictions and the measured values, the values of luminosity obtained from the theoretical predictions were compared with the measured luminosity values provided in the stellar catalog. The degree of consistency between the values was analyzed graphically and statistically. In this regard, a scatter plot was used to illustrate the degree of consistency between the theoretical predictions and the measured values of luminosity.

In addition to direct comparison, residual analysis was performed to quantify the difference between predicted and observed luminosity values. The residual for each data point is defined as

$$\text{Residual} = L_{observed} - L_{predicted}$$

Analysis of the residuals is useful for detecting systematic deviations from theoretical predictions. If the Stefan-Boltzmann relation is a good description of the data, the residuals should be randomly distributed around zero with no systematic trends [9].

In addition to this, graphical analysis was performed using the HR diagram, which plots the luminosity of the star against the effective temperature. This is an important graphical tool for the analysis of the distribution of the population of the stars and the evolutionary stage of the stars. It is possible to assess the accuracy of the theoretical relations by examining the location of the star on the diagram and the predicted and observed luminosity of the star.

Overall, the methodology employs a combination of theoretical relations from stellar physics, as well as statistical and graphical analysis of the observational data. This approach offers a quantitative framework that can be used to test the applicability of the classical relations.

Results and Discussion

- **Relationship between Temperature and Luminosity**

The first analysis deals with the correlation between the effective temperature of stars and their luminosity. A scatter plot of the luminosity of the stars against their temperature reveals an increasing trend, indicating that stars with high surface temperature also have high luminosity. This matches the theory that the amount of radiation from stars increases with temperature.

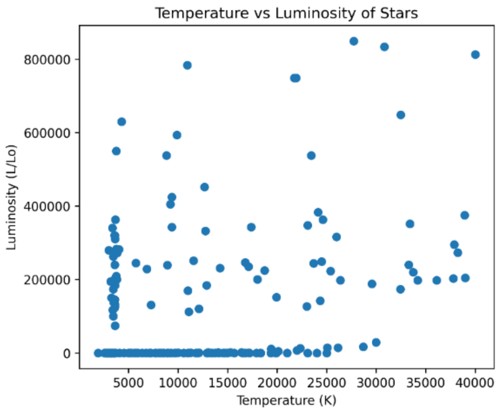

Figure 1: Scatter Plot of the Surface Temperature of the Stars Against Luminosity

As indicated in the scatter plot of the surface temperature of the stars against luminosity (fig. 1), a general positive trend is evident in the relationship between the two parameters. In general, as the surface temperature of a star increases, so does its luminosity, as predicted by the Stefan-Boltzmann Law. Although a larger cluster of data points appears in the lower temperature regime, this only indicates the larger number of cooler stars in this regime [11]. In spite of this, the general trend of the data supports the temperature-luminosity relationship as predicted in the physics of stars, as presented in the HR Diagram.

- **Verification of the Stefan–Boltzmann Relation**

To analyze the theoretical relationship between the luminosity of a star and its temperature, a logarithmic relationship was established between luminosity and temperature. According to the Stefan-Boltzmann law, the luminosity of a star is proportional to the fourth power of the effective temperature when the radius is assumed to be constant. This relationship can be expressed as $\log L \propto 4 \log T$.

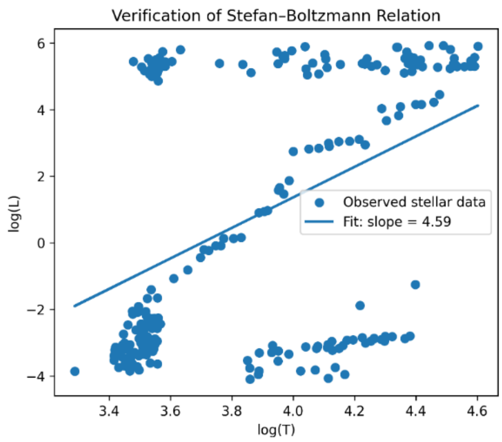

Figure 2: Log–log Plot of Stellar Luminosity Versus Effective Temperature

Fig. 2 shows the scatter plot of log L versus log T for the stars in the dataset, with the linear regression line. The slope of the fitted line is approximately 4.59, which is close enough to the expected value of 4 according to the Stefan-Boltzmann law. This shows that the observational data behaves in the expected way with respect to the scaling of temperature and luminosity.

While the overall trend follows the expected pattern according to the theory, significant scatter is observed in the distribution of the points. The reason for the scatter is the fact that the Stefan-Boltzmann law is not only dependent on the temperature of the star, and the radius of the star changes significantly for different types of stars. Stars with the same temperature may have different radii depending upon their evolutionary status [12]. Consequently, variations in stellar radius introduce dispersion in the observed luminosity values.

Despite these variations, the fitted slope close to the theoretical value verifies that the observational stellar data generally follow the expected temperature dependence of the luminosity. Similar analyses have been employed in stellar astrophysics to verify fundamental relations of radiation using observational data. [1]

- **Verification of the Stefan–Boltzmann Relation**

To check the validity of the Stefan-Boltzmann Law, the plot of luminosity against R^2T^4, where R is the radius of the star and T is the effective surface temperature of the star, was made. This equation predicts that luminosity should be directly proportional to R^2T^4.

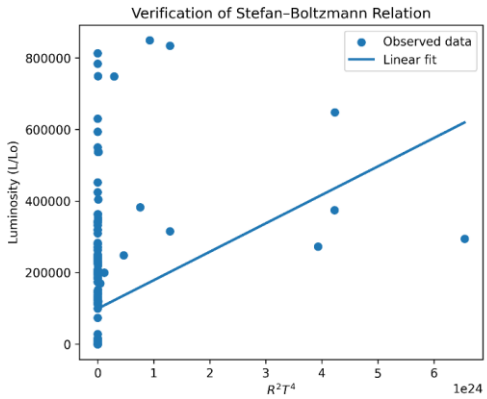

Figure 3: The scatter plot of luminosity ($L/L_\odot$) versus R^2T^4

A linear regression line was fitted to the data to examine the proportional relationship between these variables. The overall trend of the data points (fig 3) indicates that they follow a positive linear relationship. This is expected from the theoretical expectation of the Stefan-Boltzmann law. However, some level of dispersion of the data points from the line is noticed. This is expected from the possible errors in the measurement of the stellar radii and temperatures. A large number of data points are concentrated in the lower R^2T^4 region. This indicates that the data set is dominated by stars of moderate stellar radii and temperatures. The straight line that is obtained by the method of least squares is an indication of the overall trend. It is theoretically postulated by the Stefan–Boltzmann law that the luminosity versus R^2T^4 plot will pass through the origin. However, it has been observed that the line that is obtained by the method of observational data has an intercept. This is because of the uncertainties that might have been present in the data. It is still an indication of the proportionality between luminosity and R^2T^4, as is depicted by the overall trend in the plot. Therefore, the analysis provides empirical support for the Stefan–Boltzmann relation in stellar astrophysics.

- **Logarithmic Verification of the Stefan–Boltzmann Relation**

To further test the validity of the Stefan–Boltzmann Law, a logarithmic transformation of the luminosity relation was performed. Starting from the theoretical relation

$L = 4\pi\sigma R^2T^4$ L, taking the logarithm of both sides gives

$\log L = \log(4\pi\sigma) + \log(R^2T^4)$

This equation predicts a linear relationship between log L and log R^2T^4 with a theoretical slope of 1.

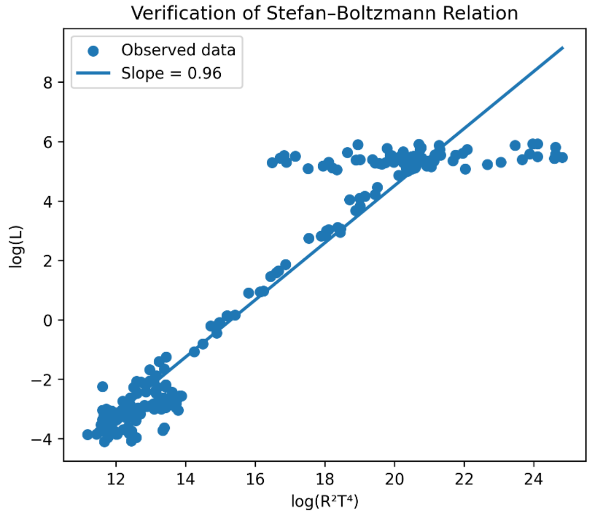

Figure 4: The Scatter plot of the Logarithmic Values of L and R^2T^4

In the scatter plot (fig 4), the logarithmic values of L and R^2T^4 for the stellar data set along with the linear regression line, has been presented. The value of the slope obtained from the linear fit has been found to be 0.96, which is very close to the expected value of 1. This strong agreement indicates that luminosity scales approximately linearly with R^2T^4 in logarithmic space.

The similarity between the observed slope and the predicted value gives strong empirical evidence for the Stefan-Boltzmann law for the description of stellar radiation. The small discrepancies from the perfect slope could be explained by the errors in the measurements of the stellar radius and temperature. In general, the logarithmic plot confirms the proportionality of the luminosity of the stars predicted by the Stefan-Boltzmann law.

- **Hertzsprung–Russell Diagram with Stellar Type Classification**

The relationship between temperature and luminosity was also explored. This was achieved by creating a log-log HR Diagram. This diagram is one of the most important tools in stellar astrophysics. To better understand the distribution of different stellar populations, the HR Diagram was plotted with different colors representing different types of stars. The luminosity (L/ L⊙) is plotted as a function of temperature T, both on logarithmic scales.

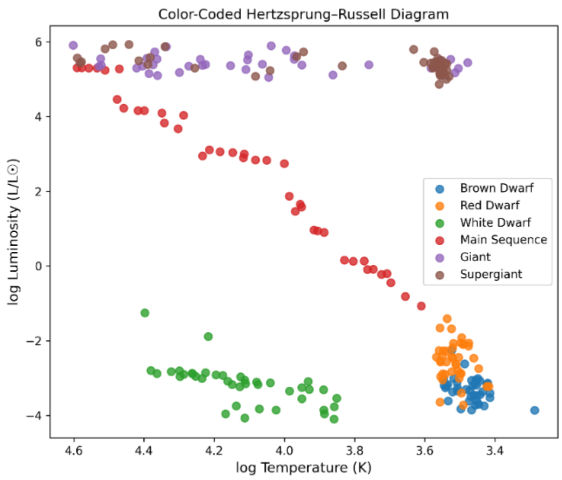

Figure 5: Scatter Plot of Stellar Log Luminosity (L/L⊙) Versus Log Temperature T

Fig. 5 shows the scatter plot of stellar luminosity (L/L⊙) versus effective temperature T in logarithmic scale. This colored diagram shows the location of the various stellar types in the HR diagram and how they are located in specific places, showing the evolutionary paths of the stars.

The distribution of points reveals several distinct regions corresponding to different classes of stars. A prominent diagonal band of stars, moving from high temperature and high luminosity down to low temperature and low luminosity, is the main sequence, in which most of the life of a star is spent converting hydrogen into helium in its core. This trend suggests that, in general, high-temperature stars tend to be high in luminosity [13]. Above the main sequence, a group of high-luminosity, though relatively low-temperature, stars can be seen, corresponding to giant and supergiant stars. The high luminosity of these stars is due to their large radii, despite their lower surface temperatures. In the lower luminosity region of the diagram, another group of stars can be found, with relatively high temperatures but very low luminosities. These stars can be classified as white dwarf stars, which are compact stellar remnants and have low radii.

The general distribution of the stars in the diagram is in conformity with the theoretical structure of the HR diagram, confirming the expected relationships between the temperature and the size of the star and the star's luminosity. The trends are in conformity with the expected theories of the dependence of the luminosity on the temperature and size of star.

- **Observed vs Predicted Luminosity**

To test the effectiveness of the theoretical luminosity relation in predicting the observed values of the luminosity of the stars, a comparison of the observed values and the predicted values of the luminosity of the stars using the Stefan-Boltzmann relation and the observed values of the radius and temperature of the stars was done. The predicted luminosity of the stars was calculated using the relation obtained from the Stefan-Boltzmann Law. A scatter plot (fig. 6) of observed luminosity versus predicted luminosity shows a strong positive correlation.

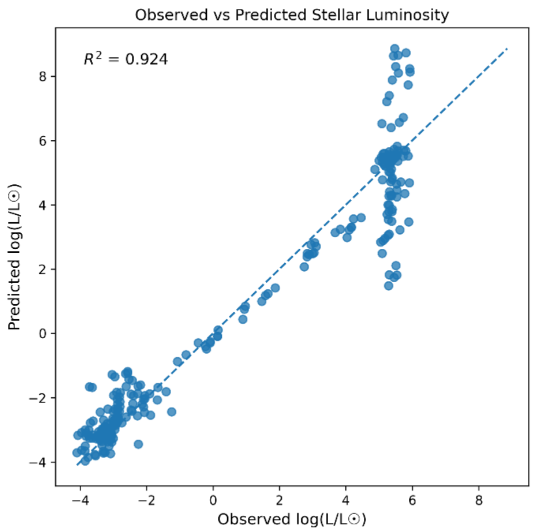

Figure 6: Observed vs Predicted Luminosity

Using the linear regression method, the coefficient of determination R^2 is found to be equal to 0.92. The value of R^2 close to unity proves that the stellar luminosity predicted using the Stefan-Boltzmann law is an excellent approximation. Small discrepancies from the perfect correlation could be attributed to various factors such as the accuracy of the measurements and errors in the observations, or the simplifying assumptions made for the stellar atmosphere models.

- **Residual Analysis by Stellar Type**

To further investigate the accuracy of the luminosity prediction, residuals were computed as the difference between observed and predicted luminosities. These residuals were then analyzed for different stellar types.

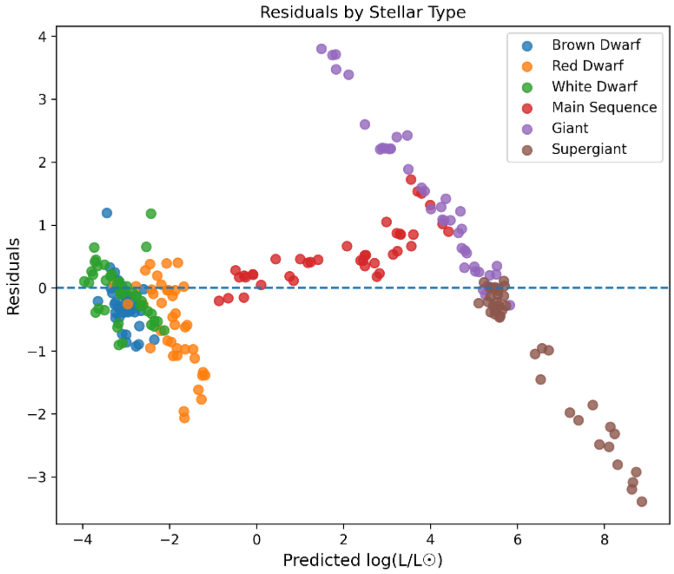

Figure 7: Residual Analysis of Predicted and Observed Luminosity for Different Stellar Types

From the residual plot (fig 7), it is observed that the residuals for main sequence stars are small. This indicates that the Stefan-Boltzmann law predicts the luminosity of main sequence stars with high accuracy. On the other hand, the residuals are large for giant and supergiant stars. This could be due to the complex atmospheres of these stars, extended envelopes, and possible variability. For white dwarfs, the residuals are also large due to the high density of these stars and compact sizes, which introduce additional physical effects not fully captured by the simplified luminosity relation [14]. From the residual plot, it is observed that the model predicts the luminosity of the stars with high accuracy. However, the residuals for a few stars deviates from the actual value.

Conclusion

In this study, observational stellar data was used to analyze the relationship between stellar luminosity, temperature, and radius, and to verify the predictions of the Stefan-Boltzmann Law. Several analyses were carried out: luminosity-temperature relationship, verification of $L \propto R^2T^4$ relationship, logarithmic regression analysis, and the HR diagram.

These results further affirm that luminosity indeed increases as the temperature and size of the star increase as predicted theoretically. This is confirmed by the logarithmic regression results that had a slope close to unity ≈ 0.96. This indicates that the results provide strong evidence for the validity of the Stefan-Boltzmann law. Moreover, the results from the comparison between observed and predicted luminosity had a high R^2 value of 0.92, which indicates that the model was successful in explaining the variation in the luminosity of the star.

HR diagram in this study has also shown the expected distribution of stellar populations, such as the main sequence, giant stars, and white dwarfs. This further proves the basic relationships between stellar temperature, luminosity, and evolutionary stages.

A significant feature of this research is that an integrated analysis approach has been used to verify the Stefan-Boltzmann relation using observational stellar data. Instead of purely theoretical discussion, an integrated approach is used that includes direct luminosity-temperature analysis, verification of the $L \propto R^2T^4$ relation, logarithmic regression modeling, and visualization using the Hertzsprung-Russell diagram. The quantitative investigation into the proportionality relationship between luminosity and R^2T^4 with the help of regression analysis and the quantitative indicators obtained from this analysis, such as the slope and coefficient of determination, serves as an empirical and quantitative validation of the Stefan-Boltzmann Law. This combined observational and statistical approach offers a clearer demonstration of how theoretical stellar radiation laws can be tested using real astronomical datasets.

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