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Application and Significance of Vedic Mathematics in Modern Era

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Abstract

The present chapter is concerned with defining and application of Vedic *sutras* described in *Atharva* Veda, one of four Indian sacred Vedas, to do fundamental operations like addition, subtraction and multiplication of numbers in field of Mathematics. Some useful *sutras* are also derived from these *sutras* which are not described in our Vedas and are more generalized than these *sutras*. Ample examples with specific techniques have also been presented to explore and explain each case or sub case. Its speedy paperless technique is helpful to make quick, accurate and intelligent decision to solve the problems with great complexity. Visual and pattern-based presentation of these *sutras* enables to perform large calculations quickly without any error. It also provides an alternative approach to do fast and tedious calculations in fraction of seconds and reduce reliance on devices like calculators etc. which helps in fostering logical and deeper understanding of numerical concepts and improves the mental math skills as well as confidence in handling numbers. The systematic pattern followed to do calculations helps to check the answers as well with a great ease. It serves multiple strategies for solving a problem and thus cultivate logical, critical and creative thinking among users. It acts as a powerful tool that bridges the gap of traditional and modern techniques and serves as an effective supplement to evolve deeper and more intuitive appreciation of mathematics. These *sutras* are very useful in preparation of various competitive exams and also have significant application in Geometry, Calculus, Algebra, Arithmetic, Trigonometric, engineering, coding and decoding methods, preparing algorithms etc. Due to its numerous applications it is being introduced in curriculum which helps to explore a holistic approach to understand mathematical problems with logic. Due to its versatility it serves as a useful tool in

academics and in professional environment. Besides that intellectual depth of ancient Indian knowledge system promotes a sense of national pride among learners.

Keywords: Puranapuranabhyam, Sankalan, Vyavkalanabhyam, Anurupyena, Ekanyunena, Purvena.

Introduction

The literal meaning of word Veda is 'fountain head and unlimited store of all knowledge'. The four sacred Indian Vedas are *Rigveda, Atharva Veda, Samveda* and *Yajurveda* and all have their significant application in respective fields. Describing the importance of Mathematics it is written in Vedas "Like the crest on the heads of peacocks, like the gems on the hoods of cobras, mathematics is at the top of Vedanga Jyotisha". The source of Vedic mathematics is *Atharva Veda*. The methods of addition, subtraction and multiplication that we practice in our primary schools are lengthy and time consuming; moreover, there is no provision to counter check in-between whether the result obtained is right or wrong. In a survey of NCERT (National council for education research and training) it is find out that more than 50% students of class V are not able to do simple arithmetical operations like addition, subtraction, multiplication, division etc. The Vedic mathematical sutras that are expressed in our Vedas are so simple and easier that even a man with little knowledge of mathematics can do large arithmetical calculations in very less time as compared to the time taken by the traditional long-time method. Moreover, by applying these Vedic mathematical *sutras* we can check the calculation whether it is right or wrong in a few seconds.

A great contribution on collection of these *sutras* can be found in work of [1-8]. Shembalkar et al. [9] presented a review on these vedic *sutras* and explained these *sutras* in an illustrative approach. Application of these *sutras* in field of architecture is presented in work of [10]. Shrama et al. [11] presented a thorough explanation of these *sutras* in an interesting and lucid manner. Kumar [12,13] draws attention of application of vedic *sutras* in machine learning and computing techniques.

Considering its vitality and applicability these *sutras* are now being taught in many prestigious institutions in England, America and Australia. Even in field of artificial intelligence NASA scientists are applying the principles. Students preparing for various competitive exams like CAT, MAT, engineering, banking examinations, professionals, teachers and businessman are reaping its benefits. The techniques described in these *sutras* to do arithmetical calculations are going to prove very helpful to software developers also in field of coding and programming.

Importance of Vedic Mathematical Sutras in the Age of Calculators and Computers

By opting Vedic Mathematical *sutras* one can do the arithmetical calculations ten times faster than the traditional longtime method. But the question arises that in this age of calculators and high speed computers what is the need of such *sutras*? The answer lies in the fact that while using such types of machines our involvement in the process becomes negligible. According to a Medical research the constant use of calculators for more than twenty years atrophies brain. If we are not having mental exercise our brain weight may increase by five percent. And if once brain weight is increased it cannot be reduced. In Vedic Mathematical *sutras* calculations are carried out mentally which provides exercise to brain and boost up confidence also.

Complementary Numbers

Two numbers are said to be complementary if their sum is 10.   The complementary numbers of digits 0 to 9 is evident from the table 1.

Table 1

Number	0	1	2	3	4	5	6	7	8	9
Complement	10	9	8	7	6	5	4	3	2	1

Now arithmetical operations like addition, subtraction and multiplication is to be explained one by one by using various Vedic mathematical *sutras* and sub *sutras*

Addition

In traditional method of addition of two numbers the carry is to be transfer from one column to another but the Vedic method for addition avoids carrying over the digit to next columns and thus saves time.

Puranapuranabhyam

This method is useful for large calculations. In this method following steps are to be taken for useful addition

- **Working Rule**
  - Spot those numbers which gives rounded numbers on pairing.
  - Rearrange the numbers and add as per the pairing of numbers.

Example: Add 26 + 59 + 394 + 66 + 11 + 14

Solution: 26 + 59 + 394 + 66 + 11 + 14

=   (26 + 14) + (59 + 11) + (394+66)

=   40 + 70 + 460

=   (40 + 460) + 70

=   500 + 70 = 570

Sankalan Vyavkalanabhyam

This Vedic sutra is combination of two Sanskrit words '*sanklan*' means addition and '*vyavkalan*' means subtraction. This method is applicable when the pair of numbers doesn't form a base i.e. multiple of 10.

- **Working Rule**
  - Split the number as a function of two or more numbers that make entire calculation easy.

Example: Add 324 + 296 + 159 + 43

Solution: (300 + 20 + 4) + (300 – 4) + (150 + 9) + (50 – 7)

= (300 + 300 + 150 + 50) + (20 + 4 – 4 + 9 – 7)

= 800 + 22

= 822

Ekadhikena Purvena

This method is more applicable than as described the previous two.

- **Working Rule**
  - Find the column-wise sum of unit digits.
  - If the sum is greater than 10, mark a bar on that number.
  - Add the excess part to the next digit of the column.
  - Write down the total number of bars to the digit next to the unit place digit and add the two.

Example: Add 6489 + 5642 + 3241

$$
\begin{array}{cccc}
6 & 4 & 8 & 9 \\
\overline{5} & \overline{6} & \overline{4} & \overline{2} \\
3 & 2 & 4 & 1 \\
\hline
4 & 2 & 6 & 2
\end{array}
$$

Solution:

Bars		1	1	1	1
Sum	1	5	3	7	2

Subtraction

The general meaning of subtraction means removing certain things from a certain group.

Subtraction is a process of finding a quantity which when added to one of the two given quantities will give the other. These quantities are respectively known as the subtrahend and minuend and the final result is called the remainder.

Here first two methods are from Veda and third Digit separator method is not explained in Veda.

Nikhilam Navatascaraman Dasatah

The meaning of this sutra is 'All from nine and last from ten'.

- **Working Rule**
  - If upper digit is greater than lower digit normal subtraction is done.
  - If upper digit is less than lower digit then finds the complement of the difference of the last digit from ten and complement of the difference of the remaining digits a from nine.
  - At last stage subtract 1 extra from that column without taking the complement

Example: 8745 - 4599 = ?

Solution:

```
    8  7  4  5
 -  4  5  9  9
    ‾‾‾‾‾‾‾‾‾‾
    4  1  4  6
```

Vinculum Method

- **Working rule**
  - Convert a digit into a *mishrank* digit by subtracting given digit from 10 and place bar over it.
  - Apply Ekanyunena Purvena and add 1 to the digit on the left.

Example: Subtract 4568568-3478989

Solution: Place a bar over each digit on the subtrahend to signify that it is a negative number.

```
    4   5   6   8   5   6   8
    -       -   -   -   -   -
    3   4   7   8   9   8   9
    ‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾
        -       -   -   -
```

Mishrank digit	1	1	1	0	4	2	1
Original digit	1	0	8	9	5	7	9

Here 'All from 9 and last from 10' formulas are used to convert the *mishrank* digit to a non *mishrank* digit. In the unit place $\overline{1}$ = 10 – 1 = 9; Ten's place $\overline{2}$ = 9 – 2 = 7; Hundred's place 4 = 9 – 4 = 5; Thousand's place 0 = 9 – 0 = 9; Ten thousand's place $\overline{1}$ = 9 – 1 = 8

Since at this stage we are out of complement; 1 is subtracted from immediate left.

Ans = 1089579

Digit Separator Method

This method is not explained is Vedas and works more effectively for all types of subtraction problems.

- **Working Rule**
  - Draw as many vertical separator lines as the number of digits.
  - Whenever the minuend at the top is less than the subtrahend at the bottom, put remainder -1.
  - Write the complement of the negative number.
  - Club the digit and its next encircled digit from left to right to reach final result.

Example: Subtract 375988 from 823479

Solution:

8	2	3	4	7	9 ------ Minuend
-	3	7	5	9	8 8 ---- Subtrahend

		-1	-1	-1	-1

5	-5	-2	-5	-1	1

Complement of each negative number

5	5	8	5	9	1

Add from left to right as

5 [-] 5 [-1] 8 [-1] 5 [-1] 9   1

5-1 / 5-1 / 8-1 / 5-1 / 9 / 1

4 / 4 / 7 / 4 / 9 / 1

447491

Multiplication

Multiplication is repeated addition. The following methods are described here for the purpose and we are to choose the best possible method in the best situation.

Antyayordashakepi

This method is of limiting application. It is valid if sum of the unit digits of multiplicand and multiplier is 10 and remaining digits are the same.

- **Working Rule**
  - Write down the product of unit digits in the right-hand part and product of remaining digits on left hand part.
  - Left-part consists of (digit at Ten's place/hundred place) x (digit at Ten's /hundred place + 1)

Example: Multiply 93 by 97

Solution: Here sum of unit digits = 3 + 7 = 10

Digit at tens place in Multiplicand and Multiplier = 9

RHS = 7 x 3 = 21

LHS = 9 x (9+1) = 90

Ans = 9021

Nikhilam Navatascaraman Dasatah

This method is applicable when multiplicand multiplier are near to base 10^n , where n is a natural number.

- **Working Rule**
    - Place multiplicand the multiplier above and below.
    - Find the deviation of multiplicand and multiplier from the base and write them next to the digit to be multiplied.
    - The final result will have no parts.

Example: Multiply 105 by 104

Solution: Both the numbers are closer to base 100

Deviation of 105 = 105 - 100 = 5

Deviation of 104 = 104 – 100 = 4

```
105    5
104    4
```
109 / 20

Ans is 10920

Sub case (a): When one of the deviations becomes negative

In such situation the RHS will be negative,

- Subtract the RHS from the base.
- Diminish the LHS by 1

Example: Multiply 122 by 98

Solution: Both the numbers are near to 100, therefore base = 100

- Deviation of 122 = 122 – 100 = 22
- Deviation of 98 = 98 – 100 = -2

```
122    22
98    -2
```
120 / - 44

120 – 1 / 100 – 44

119 / 56

Ans: 11956

Sub case (b): Adjustment of right-side digit of the product

Here two further sub cases arise

Sub sub case b 1.1: If number of digits on the right-hand side is more than the permissible limit.

The number of digits in RHS should be in accordance with base number. In such a case transfer the extreme left digit of RHS to LHS and add them.

Example: Multiply 13 by 18

Solution: Take the base = 10

Deviation of 13 = 3

Deviation of 18 = 8

```
13         3
18         8
2   1  /  2    4
= 2 3 4
```

Sub sub case b1.2: If number of digits on the right-hand side is less than the permissible limit.

The number of digits in RHS should be in accordance with base number. In such a case place digit 0 on the extreme of left digit of RHS.

Example: Multiply 989 by 995

Solution: Take the base = 1000

Deviation of 989 = 989-1000 = -11

Deviation of 995 = 995-1000 = - 5

```
989        - 11
995        - 5
984   /   55
```

984 / 055

= 984055

Sub case c: When numbers to be multiplied are not near to a common base

- **Working Rule**
  - Find deviations after choosing suitable different bases for numbers.
  - Cancel equal number of zeros of the different bases.

Example: Multiply 107 by 1008

Solution:	Number	Deviation	Base
	107	7	100
	1008	8	1000

(107 x 10) + 8 \ 56

= 107856

Anurupyena Sutra

The literal meaning of Anurupyena is 'proportionately'. This method is applicable when multiplicand and multipliers are very far from the theoretical base.

Subcase a: When left hand figure is completely divisible by the theoretical base.

- **Working Rule**
  - Choose a theoretical base and working base. Theoretical base is taken in power of 10 and working base is taken as multiple of 10.
  - Divide the left-hand figure by the quotient of theoretical base and working base.

Example: Multiply 494 by 488

Solution: Theoretical Base = 1000

Working Base = 500

Number		Deviation
494		-6
488		-12
482	/	72

$\frac{1}{2} \times 482$ / 72

241 / 072 (making RHS suitable to theoretical base)

= 241072

Sub case b: When left hand figure is not completely divisible by the theoretical base.

- **Working Rule**
  - Add the fractional part of theoretical base to the right-hand part.

Example: Multiply 251 by 252

Solution: Theoretical base = 1000

Working base = 250

Number	Deviation
251	1
252	2
253 /	2

$$\frac{1}{4} \times 253 \qquad / \qquad 002$$

$$63\frac{1}{4} \qquad / \qquad 002$$

$$63 \qquad / \qquad 250+002$$

= 63252

Ekanyuena Purvena

This sutra is applicable when multiplier contain only '9' digit.

Sub case a: If number of digits in the multiplicand is equal or less than the number of 9s in the multiplier.

- **Working Rule**
  - Subtract 1 from the multiplicand and write the result in LHS.
  - Apply Nikhilam Navatascaramam Dasatah Vedic sutra on multiplicand and write the result in RHS

Example: Multiply 89654876 by 99999999

Solution: LHS = 89654876 – 1 = 89654875

RHS = 10345124

Ans = 8965487510345124

Sub case b: If number of digits in the multiplicand is more than the number of 9s in the multiplier.

- **Working Rule**
  - Place equal number of zeroes as the numbers of 9s to the multiplicand.
  - Subtract the original multiplicand from the figure obtained in step 1.

Example: Multiply 783459 to 9999

Solution: 7 8 3 4 5 9 0 0 0 0

                        7 8 3 4 5 9

7 8 3 3 8 0 6 5 4 1

Antyayoshatakepi

This sutra is useful when the sum of the last two digits in the multiplicand and multiplier is 100 and the remaining digits are same.

- **Working Rule**
  - Multiply the last two digits and write the product on RHS.

- Multiply the remaining digits with the digit obtained by adding 1 to it, this will be the LHS of the require result.

Example: Multiply 978 by 922

Solution: Here sum of the last two digits of the multiplier and multiplicand is 78 + 22 = 100.

LHS = 9 x 10 = 90

RHS = 78 x 22 = 1716

Ans = 901716

Vamanlyayoh Dasake Api

This sutra is applicable when the sum of digits placed at the ten's place in the multiplicand and multiplier is 10 and unit digit of both multiplicand and multiplier is the same.

- **Working Rule**
  - Consider the product of two left digits whose sum is ten and 1 to it. This is the LHS of the desired product.
  - RHS will be the square of Unit digit.

Example: Multiply 86 by 26

Solution: Here sum of ten's digit of multiplicand and multiplier is 8 + 2 = 10

LHS = Product of digits at ten's place + unit digit

= (8 x 2) + 6

= 22

RHS = Square of unit digit

= 6 x 6 = 36

Ans = 2236

Vamanlyayoh Dasake Gunijah Api

This sutra is useful when the sum of the left two digits other than the unit is a multiple of ten and unit digits are the same.

- **Working Rule**
  - The left-hand side of the desired result is derived from the formula   10 + sub base x unit digit
  - RHS = square of unit digit

Example: Multiply 278 by 238

Solution: Here sum of the left digits in multiplicand and multiplier is 27 + 23 = 50, which is a multiple of 10.

50 = 5   x   10

Sub base   Base

LHS = Product of two left digits whose sum is a multiple of 10 + sub base x unit digit

= 27 x 23 + 5 x 8 = 661

RHS = Square of unit digit
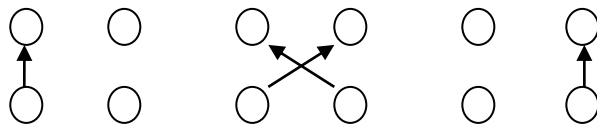
= 8 x 8 = 64

Ans = 66164

Urdhva Tiryagbhyam

The meaning of 'Urdhva Tiryagbhyam' is 'vertically and cross-wise'.

This formula is more relevant and more generalized than all the formulas as described above.
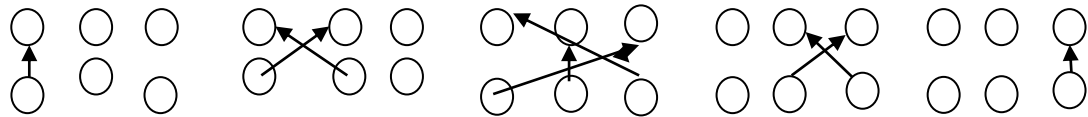
- **Working Rule**

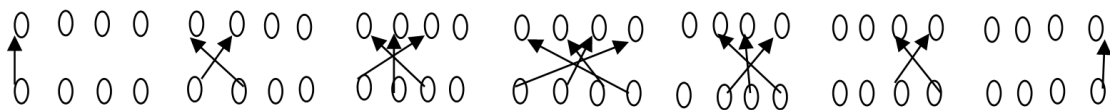This method can be understood by dot and stick method as explained further.

Multiplication of two-digit numbers

Multiplication of three digit numbers

Multiplication of four digit numbers

The same process can be opted to multiply numbers of five or more digits.

Example: Multiply numbers 8989 and 8892

Solution: 8   9   8   9

                    8   8   9   2

6 4 / 1 3 6 / 2 0 8 / 2 3 3 / 1 6 2 / 9 7 / 1 8

=   6   4   6   8   3   2   7   8

              13   20   23   16   9   1

         7   9   9   3   0   1   8   8

Multiplication of three numbers by Vedic method

In Vedic method the multiplication of numbers can be done easily with the help of base or sub case as is explained with the help of different cases and relevant examples.

Case 1. When numbers to be multiplied are nearer to base 10^n.

- **Working Rule**
  - Write the deviation of each number from its base.
  - Add all the deviations to the working base.
  - Make all the possible permutations of the deviation taking two at a time and add them.
  - Multiply deviations.
  - Arrange the result obtained in last three steps.

Example: Multiply 105 x 104 x 109

Solution: The working base = 100

Number	Deviation
105	5
104	4
109	9

Sum of base and deviations = 100 + 5 + 4 + 9 = 118

Multiply the deviations in pairs of two and sum up, the result obtained is

5 x 4 + 4 x 9 + 5 x 9 = 101

Multiply the deviations, 5 x 4 x 9 = 180

Arrange the result of all the above steps in a vertical separator and add them up from left to right

118 / 101 / 180

= 1190280

Case 2: When the working base is 50, 500, 5000………

After following steps as mentioned in case 1, proceed as mentioned below

Write the last result in column.

Keep the right hand figure of column three intact and divide the figure in the 2nd column by ½ and the figure in the 1st column by ¼.

Add the respective digits of each column from left to right.

Examples: Multiply 54 x 48 x 61

Solution: Working base = 50 = 100 / 2

Number	Deviation
54	4
48	-2
61	11

Base + deviations = 50 + 4 -2 + 11= 63

Multiply the deviations in pairs of two and sum up the results so obtained

4 x (-2) + 4 x 11 + 11 x (-2) = 14

Multiply the deviations

4 x (-2) x 11 = -88

Final result is 63 / 14 / -88

Write the result in columns

Column 1	**Column 2**	**Column 3**
63 x ¼	14 x ½	-88
15 + ¾	7	-88
15	75 + 7	-88
15	82	100-88
15	81	12

Ans = 158112

Multiplication of four numbers by Vedic method

Following steps are to be followed to find the product of four numbers by vedic method

- **Working Rule**
  - Select suitable base and find the deviation of each number.
  - Find sum of any number and other three deviations.
  - Find sum of product of three numbers at a time.
  - Find product of all deviations.

Example: Multiply 995 x 996 x 997 x 998

Solution: Consider the base = 1000

Number	Deviation
995	-5
996	-4
997	-3
998	-2

Sum of any number and other three deviations

= 995 – 4 – 3 – 2 = 986

Sum of product of two deviations

= (-5) x (-4) + (-5) x (-3) + (-5) x (-2) + (-4) X (-3) + (-4) x (-2) + (-3) x (-2) = 71

Sum of product of three numbers at a time

= (-5) x (-4) x (-3) + (-5) x (-4) x (-2) + (-4) x (-3) x (-2) + (-5) x (-3) x (-2) = -154

Product of all deviations

= -2 x -3 x -4 x -5 = 120

Required form is 986 / 71 / -154 / 120

$\qquad$ = 986 / 070 / 1000-154 / 120

$\qquad$ = 986 / 070 / 846 / 120

$\qquad$ Ans $\quad$ = 986070846120

Case 1. Multiplication of four numbers when the base is a multiple of 100, 1000…..

- **Working Rule**
    - Select a suitable base and sub-base.
    - Calculate the first part of the required result using formula
    - (Sum of any number and other three deviations) x (sub – base).
    - 2nd part is evaluated using formula (Sum of product of two deviations) x (sub-base)^2.
    - Find 3rd part using formula (sum of product of three numbers at a time x (sub-base).
    - 4th part = product of all deviations.

Example: Evaluate 506 x 507 x 508 x 509

Solution: Here all the numbers to be multiplied are near to 500.

$\qquad$ Therefore Base = 100, and sub-base = 5. Deviations of the number from 500 are given below

Number	Deviation
506	6
507	7
508	8
509	9

Ist part = (Sum of any number and other three deviations) x (Sub-base)^3

$\qquad$ = (506 + 7 + 8 + 9) x 5^3 = 530 x 125 = 66250

2nd part = (Sum of product of two deviations) x (Sub-base)^2

$\qquad$ = (6 x 7 + 6 x 8 + 6 x 9 + 7 x 8 + 7 x 9 + 8 x 9) x 5^2 = 335 x 25 = 8375

3rd part = (Sum of product of three numbers at a time) x (sub-base)

= (6 x 7 x 8 + 6 x 7 x 9 + 6 x 8 x 9 + 7 x 8 x 9)x 5

= 1650 x 5 = 8250

4th part = Product of all deviations

= 6 x 7 x 8 x 9 = 3024

Hence 506 x 507 x 508 x 509 = 66250 / $_{83}$75 / $_{82}$50 / $_{30}$24

Since the base = 100, each part will contain maximum two digits and the rest will be transferred to the next part. The excess digit is written in subscript.

506 x 507 x 508 x 509 = 66334578024

Conclusion

From the various methods described above to do arithmetical calculations it is evident that Vedic mathematics presents a lucid and unconventional methodology to do these calculations. Very tedious and cumbersome mathematical calculations can be carried out within seconds and in a playful manner. The accuracy of the result can be judged in seconds; moreover, it is ten times faster than the traditional method that we opt in the classrooms. Thus by opting this it is certainly going to remove the "math-phobia" many students suffer from. It is up to us how deep we jump in to the ocean of Vedic *sutras* and collect the pearls of our choice.

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